

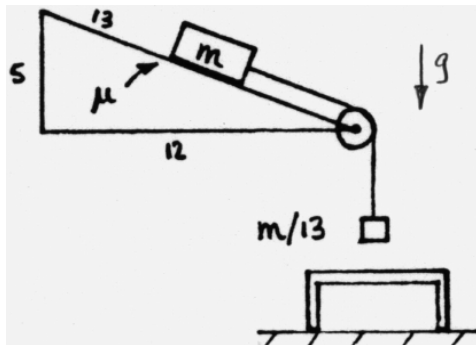
# EXAMINATION 1

**Directions.** Do all four problems (weights are indicated). This is a closed-book closed-note exam except for one  $8\frac{1}{2} \times 11$  inch sheet containing any information you wish on both sides. You are free to approach the proctor to ask questions – but he or she will not give hints and will be obliged to write your question and its answer on the board. Don't use a calculator, which you don't need – roots, circular functions, *etc.*, may be left unevaluated if you do not know them. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Box or circle your answer.

1. (20 points) You wish to fly (at low altitude) from Sapporo, Japan to Portland, OR. These cities are both at  $45^\circ$  north latitude (halfway between the equator and the north pole), and they are separated by  $90^\circ$  in longitude (azimuth). Consider the earth to be a sphere of radius  $R$ .

- (10 points) You fly a course that keeps you at constant latitude, *i.e.* you fly due east. Over what distance do you travel?
- (10 points) You fly a “great circle” course that takes you from Sapporo to Portland in the shortest possible distance (without penetrating the earth, of course). What is that distance?

2. (30 points) A block of mass  $m$  slides on an inclined plane with a slope of  $5/12$  (*i.e.* the slope of the hypotenuse of a 5-12-13 triangle). A massless rope, guided by a massless pulley, connects the block to a second block of mass  $m/13$ , which is hanging freely above a lower table.



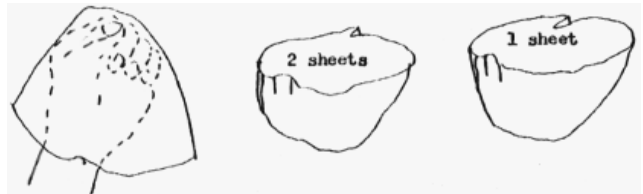
- (15 points) The two-block system is observed to be moving with a constant velocity  $v_0$ . What is the coefficient  $\mu$  of sliding friction between block and plane?

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- (15 points) After the hanging block hits the table, what is the distance  $s$  along the surface of the plane along which the top block continues to slide? (Assume that the plane is long enough that the block does not fall off. If you are unsure of your answer to part (a.), you may leave  $\mu$  as an undetermined constant.)

3. (35 points) To determine the dependence of the force of air resistance upon the speed of a slowly moving object, one starts with three sheets of paper stacked together as they come out of the package. One then crumples them against one's fist as shown in the sketch.

Next one separates a single sheet from the other two without changing their crumpled (pseudo-conical) shapes. This yields two objects with the same shape, but with different masses  $m$  and  $2m$ , where  $m$  is the mass of a single sheet.



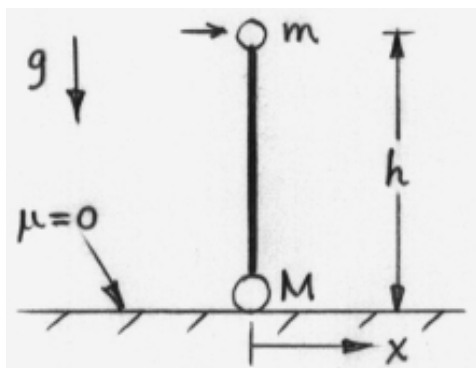
Finally one releases these two objects simultaneously, and compares their motion in still air under the influence of gravity.

- (5 points) Instantaneously after the two objects are released, what is the ratio  $R$  of

the (downward) acceleration of the heavier object to that of the lighter object?

- b. (15 points) Very soon after being released, the two objects reach terminal (constant) velocity due to the effects of air resistance. After a long time, one observes that the object of mass  $2m$  has dropped  $\sqrt{2}$  times farther than the object of mass  $m$ . Assuming that the force of air resistance on these objects is proportional to  $v^\alpha$ , where  $v$  is the velocity and  $\alpha$  is a constant exponent, what is the value of  $\alpha$ ?
- c. (15 points) Suppose that Mother Nature were to turn gravity off while these objects are falling. If one were willing to wait an arbitrarily long time, would they fall an arbitrarily long distance, or would that distance be bounded? Explain.

4. (15 points). An asymmetric barbell stands vertically at rest on frictionless horizontal ice. Mass  $M$  rests on the ice, and mass  $m$  is a distance  $h$  above it; the mass of the bar that rigidly connects these two masses is negligible. The dimensions of the masses can be neglected in comparison to their separation  $h$ . Take  $x = 0$  to be the initial position of mass  $M$ .



Mass  $m$  is given an infinitesimal tap in the  $+\hat{x}$  direction that produces a negligible momentum, but does eventually cause the barbell to topple over. You may assume that mass  $M$  remains in contact with the ice throughout the motion. When mass  $m$  hits the ice, at what horizontal coordinate  $x_M$  will mass  $M$  be located?